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Technical Note

An experiment of state estimation for predictive maintenance using Kalman filter on a DC motor

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Abstract

Preventive maintenance (PM) is an effective approach to promoting reliability. Time-based and condition-based maintenance are two major approaches for PM. No matter which approach is adopted for PM, whether a failure can be early detected or even predicted is the key point. This paper presents the experimental results of a failure prediction method for preventive maintenance by state estimation using the Kalman filter on a DC motor. The rotating speed of the motor was uninterruptedly measured and recorded every 5 min from 1 April until 20 June 2001. The measured data are used to execute Kalman prediction and to verify the prediction accuracy. The resultant prediction errors are acceptable. Futhermore, the shorter the increment time for every step used in Kalman prediction, the higher prediction accuracy it achieves. Failure can be prevented in time so as to promote reliability by state estimation for predictive maintenance using the Kalman filter.

Keywords: Kalman filter; State estimation; Failure prediction; Preventive maintenance; DC motor

1. Introduction

High quality and excellent performance of a system are always goals for engineers to achieve. Reliability engineering integrates quality and performance from the beginning to the end of a system life [1]. Therefore, reliability can be treated as the time dimensional quality of a system. Reliability is affected by every stage throughout the system life, including its development, design, production, quality control, shipping, installation, operation, and maintenance. Consequently, paying attention to each of the stages can promote reliability. Specifically, in the onsite operation phase, failures are the main causes that worsen performance and degrade reliability. Accordingly, failure avoidance is the main approach to reliability assurance. There are three main types of maintenance, namely improvement maintenance (IM), preventive maintenance (PM), and corrective maintenance (CM) [2]. The efforts of IM are to reduce or eliminate entirely the need for maintenance, i.e. IM is performed at the design phase of a system emphasizing elimination of failures that require maintenance. There are many restrictions for a designer, however, such as space, budget, market requirements, etc. Usually, the reliability of a product is related to its price. By contrast, CM is the repair actions executed after failure occurrence. PM denotes all actions intended to keep equipment in good operating condition and to avoid failures [2]. As a result, PM should be able to pinpoint when a failure is about to occur, so that repair can be performed before such failure causes damage.

PM is an effective approach to promoting reliability [3]. Time-based and condition-based maintenance are two major approaches for PM. No matter which approach is adopted for PM, whether a failure can be early detected or even predicted is the key point. If a device is judged to know that it is going to fail by the predicted future state variables, the failure can be prevented in time by PM. However, future state variables should be accurately predicted at a reasonably long time ahead of failure occurrence [4,5]. A failure prediction study entitled 'State estimation for predictive maintenance using Kalman filter' has been proposed [6]. In the study, failure times were generated by Monte Carlo simulation and predicted by the Kalman filter. One-step-ahead and two-step-ahead predictions were conducted. Resultant prediction errors are sufficiently small in both predictions. Even so, the failure prediction was simulated on a computer after all. In the current study, a DC motor and a data acquisition system are set to implement the simulation. The rotating speed of the motor is chosen as the major state variable to judge whether the motor is going to fail by state estimation using the Kalman filter. The rotating speed of the motor was uninterruptedly measured and recorded

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Nomenclature

$(\cdot)_k$	The value of $(\cdot)$ at time kT
$(\hat{\cdot})_{a/b}$	The estimate of $(\cdot)$ at time aT based on all known information about the process up to time bT
A	A matrix
A_c	Coefficient matrix of the state equation for a continuous system
A_d	Coefficient matrix of the state equation for a discrete system
A^T	Transpose matrix of A
A^{-1}	Inverse matrix of A
B	Damping coefficient
B_c	Coefficient matrix of the state equation for a continuous system
B_d	Coefficient matrix of the state equation for a discrete system
B_k	Coefficient matrix for the input term of a discrete state equation
C	A matrix
C_c	Coefficient matrix of the state equation for a continuous system
C_d	Coefficient matrix of the state equation for a discrete system
D_c	Coefficient matrix of the state equation for a continuous system
D_d	Coefficient matrix of the state equation for a discrete system
E	Applied voltage
E_r	Estimation error
H_k	Matrix giving the ideal (noiseless) connection between the measurement and the state vector
i_a	Armature winding current
J	Moment of inertia of rotor and load
k_b	Back emf constant
K_k	Kalman gain
k_T	Motor torque constant
L_a	Armature winding inductance
L^{-1}	The inverse Laplace transform
$P_{k/k-1}$	Estimation error covariance matrix
Q_k	Covariance matrices for disturbance
R	Armature winding resistance
R_k	Covariance matrices for noise
t	Time variable
T	Motor output torque
T	Increment time for every step in Kalman prediction
U_k	Control input of a discrete state equation at state k
V	Variation of the estimated rotating speed
V_k	Noise, measurement error vector. It is assumed to be a white sequence with known covariance
W_k	Disturbance, system stochastic input vector. It is assumed to be a white sequence with known covariance and having zero cross-correlation with V_k sequence
x, X	Variable of a distribution function
X_{D0}	Initial states resulting from deterministic input
X_k	System state vector at state k
X_{S0}	Initial states resulting from stochastic input
Y_k	System output vector at state k
Z_k	Output measurement vector
θ	Motor angle displacement
$\dot{\theta}$	Motor rotating speed
μ	Mean value of a distribution function
σ	Standard deviation of a distribution function
Φ_k	Matrix relating X_k to X_{k+1} in the absence of a forcing function. It is the state transition matrix if X_k is sampled from a continuous process

every 5 min from 1 April until 20 June 2001. Instead of simulated data, the measured data are used to execute Kalman prediction and to verify the prediction accuracy in the current study.

In the next section, equations for state estimation of the Kalman filter are briefly introduced. Section 3 presents the transfer function, continuous state model, and the discrete state model of a DC motor that is employed in this paper. Section 4 presents the experiment setup with related parameters. Results and discussions are in Section 5.

2. Kalman filtering

The block diagram of a discrete system is shown in Fig. 1. The state equations [7] are:

$$X_{k+1} = \Phi_k X_k + B_k U_k + W_k, \tag{1}$$

$$Y_k = H_k X_k, \tag{2}$$

$$Z_k = Y_k + V_k. \tag{3}$$

State estimation aims to guess the value of X_k by using measured data, i.e. Z_0, Z_1, … Z_{k-1}. Let $a \geq b$, and define the notation $(\hat{\cdot})_{a/b}$ as the estimate of $(\cdot)$ at time aT based on all known information about the process up to time bT. Accordingly, $\hat{X}_{k/k-1}$ is called the prior estimate of X, and $\hat{X}_{k/k}$ is called the posterior estimate of X [8].

The Kalman filter is a copy of the original system and is driven by the estimation error and the deterministic input. The block diagram of the filter structure is shown in Fig. 2. The filter is used to improve the prior estimate to be the posterior estimate by the measurement Z_k. A linear blending of the noisy measurement and the prior estimate is written as given in Ref. [8]

$$\hat{X}_{k/k} = \hat{X}_{k/k-1} + K_k(Z_k - H_k \hat{X}_{k/k-1}), \tag{4}$$

where K_k is a blending factor for this structure. As depicted

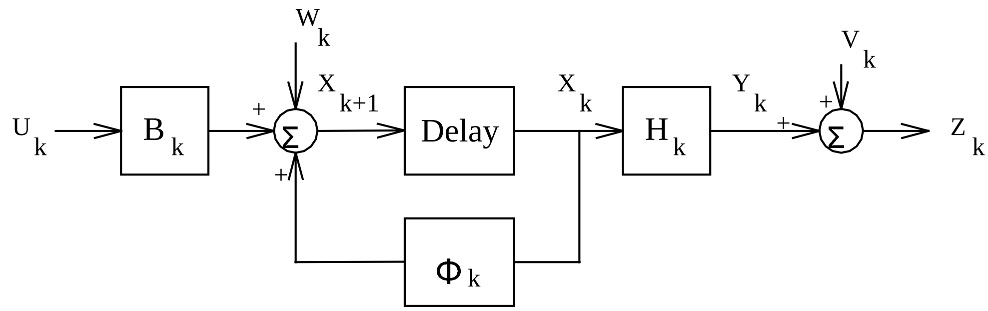


Fig. 1. Block diagram of a discrete system.

in Fig. 2, the one-step-ahead estimate is formulated as

$$\hat{X}_{k+1/k} = \Phi_k \hat{X}_{k/k-1} + \Phi_k K_k (Z_k - H_k \hat{X}_{k/k-1}) + B_k U_k$$
$$= \Phi_k [\hat{X}_{k/k-1} + K_k (Z_k - H_k \hat{X}_{k/k-1})] + B_k U_k$$
$$= \Phi_k \hat{X}_{k/k} + B_k U_k. \quad (5)$$

According to the aforementioned statements, recursive steps for constructing an one-step estimator are summarized in Fig. 3. However, initial conditions, i.e. $\hat{X}_{0/-1}, P_{0/-1}, \Phi_0, H_0, Q_0$, and R_0 have to be known to start recursive steps.

3. Armature-controlled DC motor

An armature-controlled DC motor is employed in this study to perform state prediction. The motor circuit representation is shown in Fig. 4. The transfer function of a DC motor is derived as [9]

$$\frac{\theta(s)}{E(s)} = \frac{k_T}{s[(sL_a + R)(sJ + B) + k_T k_b]}. \quad (6)$$

Define θ, $\dot{\theta}$, and i_a as state variables, so that the state vector is $X = [\theta \quad \dot{\theta} \quad i_a]^T$, where

$$\frac{d}{dt}\theta = \dot{\theta}. \quad (7)$$

In measurement, the rotating speed $\dot{\theta}$ is the motor output. Accordingly, the continuous state equations of the DC motor are

$$\frac{d}{dt}\begin{bmatrix} \theta \\ \dot{\theta} \\ i_a \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{B}{J} & \frac{k_T}{J} \\ 0 & -\frac{k_b}{L_a} & -\frac{R}{L_a} \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix} E, \quad (8)$$

$$Y = [0 \quad 1 \quad 0]\begin{bmatrix} \theta \\ \dot{\theta} \\ i_a \end{bmatrix}. \quad (9)$$

The general form of state equations for a continuous system reads [10]:

$$\dot{V}(t) = A_c V(t) + B_c U(t), Y(t) = C_c V(t) + D_c U(t). \quad (10)$$

Let $\Phi_c(t) = L^{-1}[(sI - A_c)^{-1}]$ be the state transition matrix for Eq. (10), where L^{-1} denotes the inverse Laplace transform. The discrete state equations sampled from

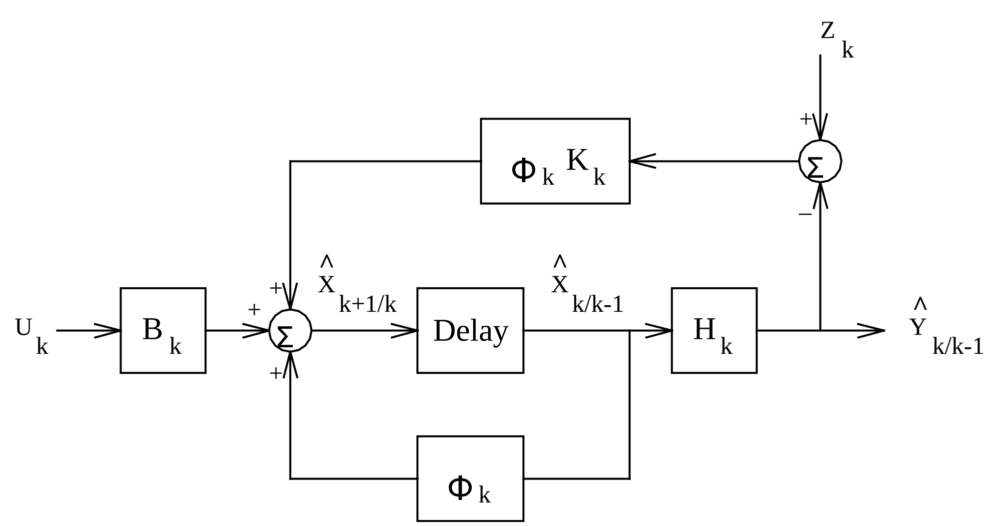


Fig. 2. Block diagram of Kalman filter.

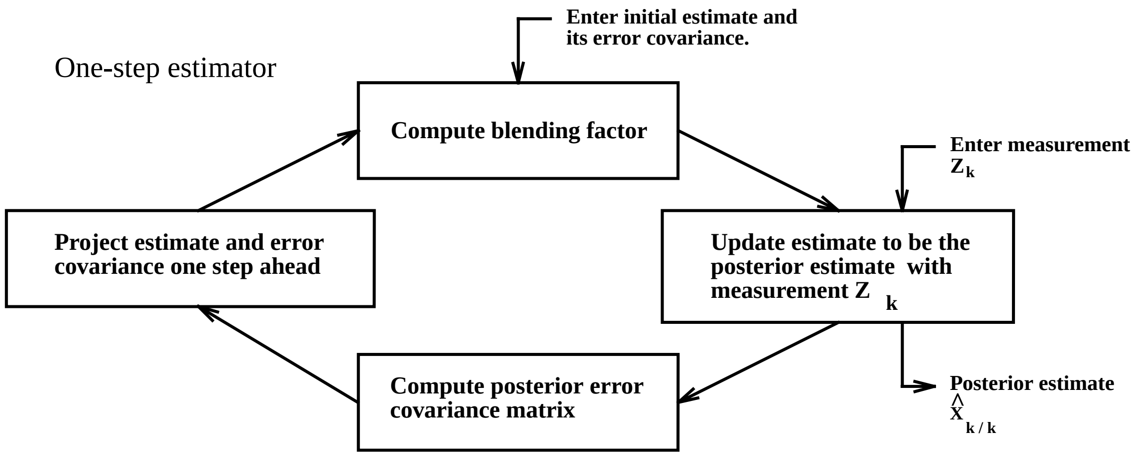


Fig. 3. One-step estimator.

Eq. (10) by a sample-and-hold with time interval T seconds are as follows [11]:

$X_k + 1 = A_d X_k + B_d U_k,$

$Y_k = C_d X_k + D_d U_k,$

where

$$A_d = \Phi_c(T), \tag{11}$$

$$B_d = \left[\int_0^T \Phi_c(\tau)\mathrm{d}\tau\right] B_c, \tag{12}$$

$$C_d = C_c, \tag{13}$$

$$D_d = D_c. \tag{14}$$

4. Experiment setup

The experiment setup, as shown in Fig. 5, is composed of a DC motor with driver unit and a data acquisition system.

4.1. DC motor

The DC motor used in this experiment is made by TECO, Taiwan. The model number of the motor is GSDT-1/2 hp. Parameters for the DC motor used in this study are as follows [12]: $E = 150$ V, $B = 0.001135$ N m s, $J = 0.0102$ kg m^2, $K^T = 0.153$ N m/A, $K_b = 1.926$ Vs , $R = 3.84\ \Omega$, $L_a = 0.01$ H. Substituting them into Eqs. (8) and (9), the continuous state equations of the motor become

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\theta\\ \dot{\theta}\\ i_a\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & -0.111 & 15\\ 0 & -192.6 & -384\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ i_a\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 100\end{bmatrix}150, \tag{15}$$

$$Y = [0 \quad 1 \quad 0]\begin{bmatrix}\theta\\ \dot{\theta}\\ i_a\end{bmatrix}. \tag{16}$$

The discrete state equations sampled from Eqs. (15) and (16) with time interval T = 1200 s are

$$\begin{bmatrix}\theta_{k+1}\\ \dot{\theta}_{k+1}\\ i_{a,k+1}\end{bmatrix} = \begin{bmatrix}1 & 0.13098 & 0.0051164\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta_k\\ \dot{\theta}_k\\ i_{a,k}\end{bmatrix} + \begin{bmatrix}613.91\\ 0.51164\\ 0.0037955\end{bmatrix}150, \tag{17}$$

$$Y_k = [0 \quad 1 \quad 0]\begin{bmatrix}\theta_k\\ \dot{\theta}_k\\ i_{a,k}\end{bmatrix}. \tag{18}$$

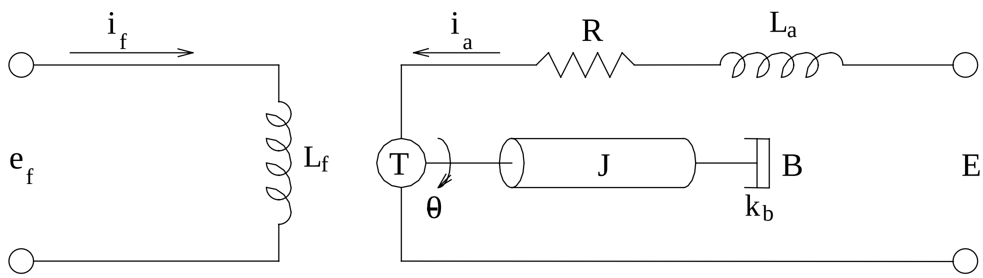
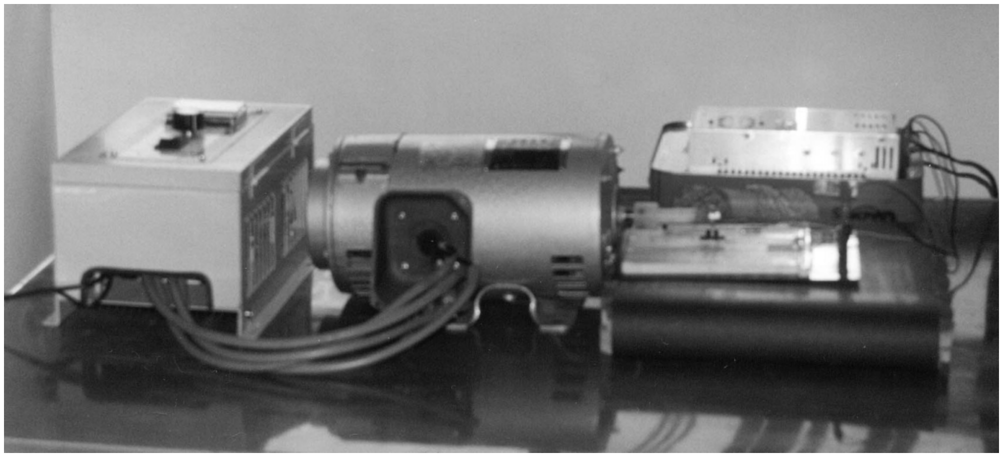


Fig. 4. Circuit representation of DC motor.



(a)

(b)

Fig. 5. Experiment setup.

The following parameters are also used to conduct state estimation in this study.

1. Sampling interval T = 20 min that is the increment time for every step in Kalman prediction. For comparing results among shorter and longer T, this study performed another two estimations with different time intervals between two states, i.e. T = 5 and 60 min.
2. Disturbance W_k has mean 0 and variance 0.1 V [13].
3. Measurement error V_k for $\dot{\theta}$ has zero mean and 1% full-scale accuracy [14] of the measurement.
4. The rated rotating speed of the DC motor is 3180 rpm [12], which is prescribed as the initial value of the state variable $\dot{\theta}$.

4.2. Data acquisition system

The data acquisition system used in this study is composed of a photo-interrupter circuit, a personal computer (PC), and a RS-232 transmission interface [14]. The rotating speed of the DC motor is measured by the photo-interrupter coded GP1S02. The shaft of a rotary disk is connected to the shaft of the DC motor, and the disk is placed between the light-emitting element and the light-receiving element of the photo-interrupter so as to generate pulse-signals while the motor rotates. The device and the circuit are shown in Fig. 6.

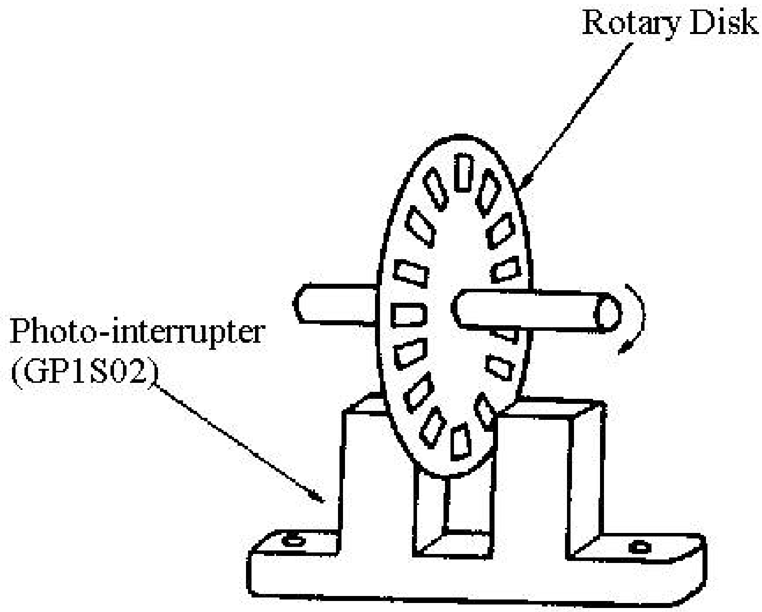
Pulse-signals are transmitted to the PC through the RS-232 interface, and the PC counts the pulses that are accumulated within 60 s in order to derive the rotating speed in rpm (revolutions per minute).

5. Results and discussions

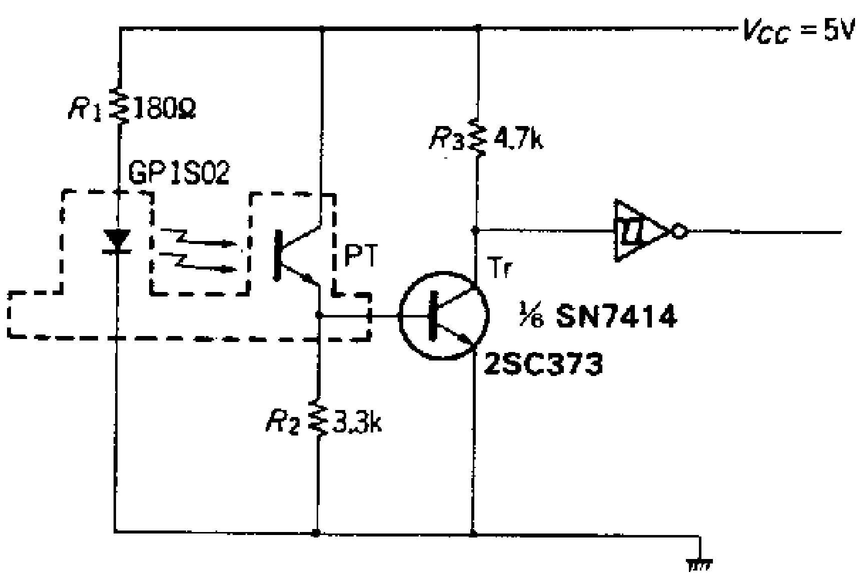
5.1. Results

5.1.1. Measured data

The rotating speed of the motor was measured and recorded every 5 min day and night from 1 April until 20 June 2001. Because the experiment lasted for nearly three months, a large number of data were thus accumulated. There are 288 measurements in a day and 23,040 data in total for that period of time. Fig. 7 shows the results. The



(a)



(b)

Fig. 6. Device and circuit for rotating speed measurement.

data were fed into the estimator, as depicted in Fig. 3, to estimate the one-step-ahead state variables. The measured data and the resultant estimates for T = 20 min (i.e. every four measurements) are shown in Fig. 8. The data will be mixed up and become hard to read if all 23,040 data are shown in one chart. To avoid this and to present the results more clearly, the time-axis unit of Fig. 8 is set to be 24 h (i.e. one point per day).

5.1.2. Estimate error percentage

The estimate error percentage is defined as

$$E_r\% = \frac{\hat{\dot{\theta}}_{k+1/k} - \dot{\theta}_{k+1}}{\dot{\theta}_{k+1}} \times 100\%. \tag{19}$$

E_r represents the difference between predicted value and

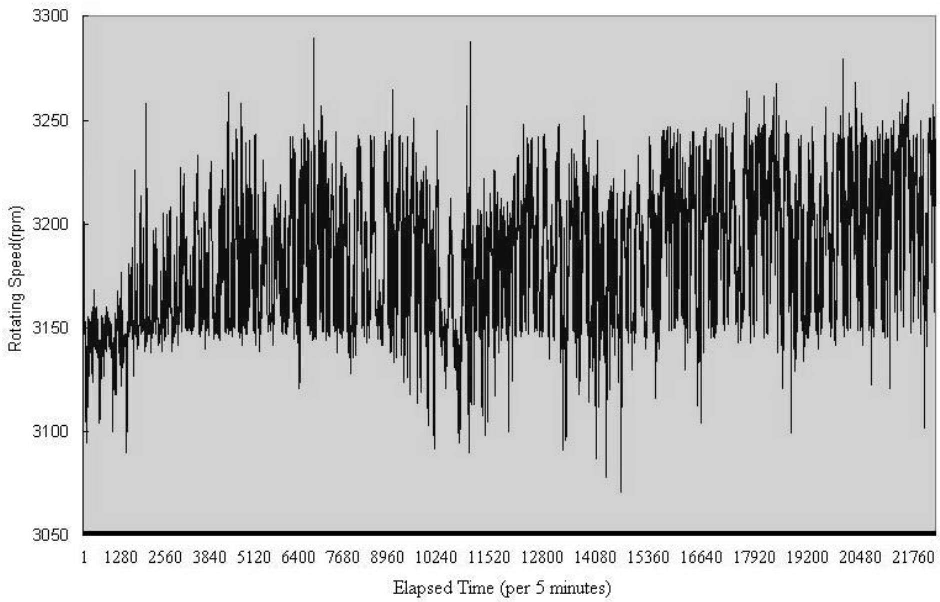


Fig. 7. Measured rotating speed for every 5 min.

actual value. Fig. 9 shows the results that are derived from Eq. (19) using the data in Fig. 8. Reading from Fig. 9, the maximum $E_r\%$ is less than 3%.

5.1.3. Mean value and variance of the estimate accuracy

Let

$$X_i = 1 - E_r\%_i, i = 1, 2, \ldots, 23,040 \tag{20}$$

be the individual accuracy of each estimate, and

$$\mu = \frac{\sum_{i=1}^{23,040} X_i}{23,040}, \tag{21}$$

$$\sigma^2 = \frac{\sum_{i=1}^{23,040} (X_i - \mu)^2}{23,040} \tag{22}$$

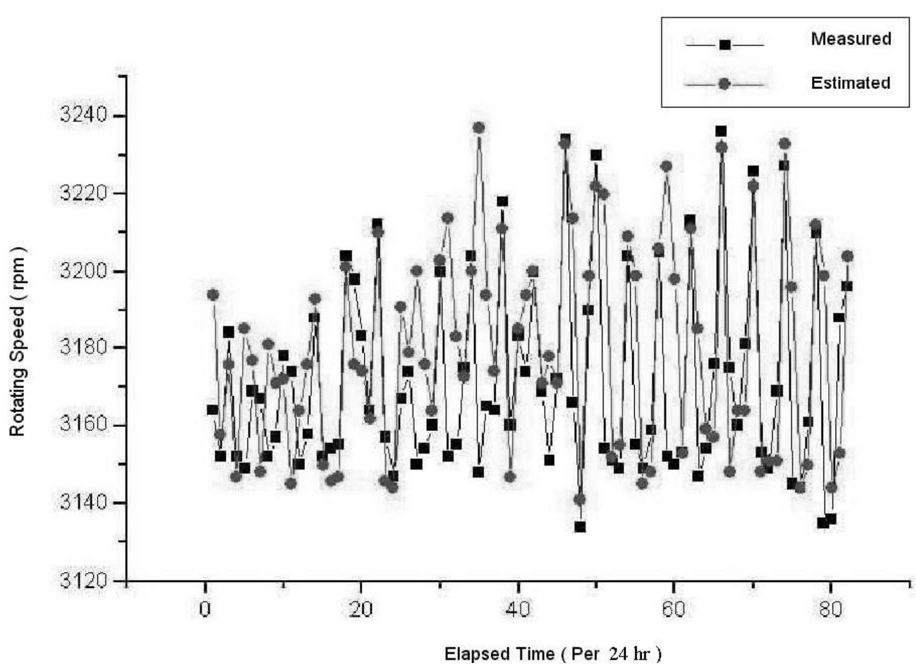


Fig. 8. Measured and estimated rotating speed of the motor.

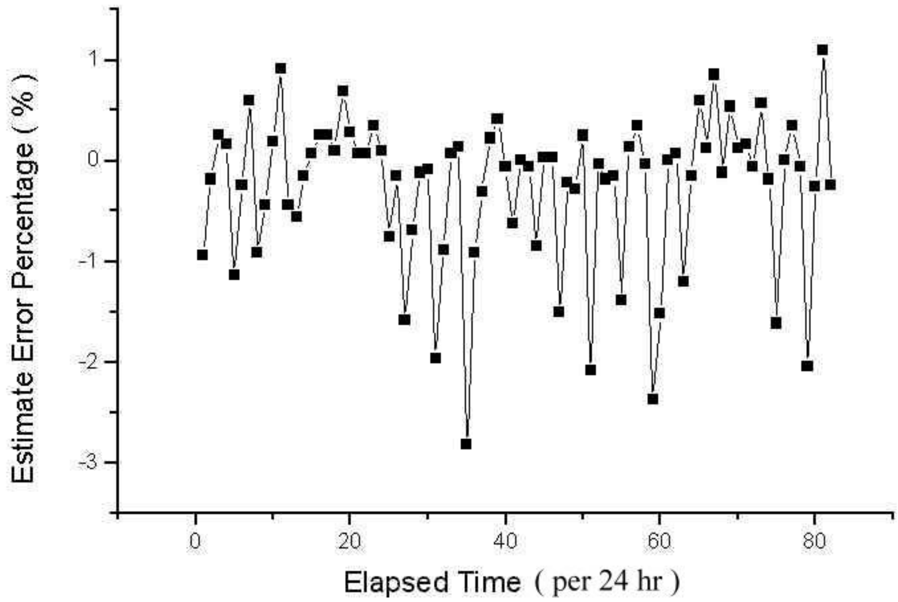


Fig. 9. Estimate error percentage.

be the mean value and the variance [15] of the accuracy for the 23,040 samples, respectively. According to Eqs. (20)–(22), the resultant mean values, variances, and standard deviations (σ) of the estimate accuracy for $T = 5$, 20, and 60 min are summarized in Table 1.

5.1.4. Rotating speed variation

Variation percentage of the estimated rotating speed of the DC motor is defined as

$$V\% = \frac{\hat{\hat{\theta}}_{k+1/k} - 3180}{3180} \times 100\%. \tag{23}$$

$V\%$ represents the variation percentage of the estimated rotating speed from the rated value 3180 rpm, i.e. the abnormality extent of the motor performance. It is used to judge whether the motor is going to fail or not. Since the mean time between failure (MTBF) of a motor is about 100,000 h [16], the rotating speed of the motor in this study varied less than 2% of the rated value during the experiment time period. Variation percentage of the estimated rotating speed of the DC motor is shown in Fig. 10.

5.2. Discussions

1. The mean estimate accuracies for $T = 5$, 20, and 60 min are all higher than 99.7%, which infers that the one-step-ahead state variable can be accurately predicted using the proposed method in this experiment.
2. A threshold is a value used to judge an equipment failure occurs or not. It is prescribed as the measurement value that is taken just prior to or at the time of failure [17]. For failure prediction, the threshold for a motor should be determined by the user of the motor according to requirements for specific situations. Once the estimated value reaches the threshold, the failure is predicted.
3. The disturbance amplitude should be composed of all possible uncertainties of the motor and the environment.
4. Since the prediction is for PM purpose, the prediction time should be reasonably long enough for the PM action.
5. The proposed method in this study is exemplified by a motor system, which is treated as a component. The procedure can be executed on a multi-component system if state equations for the components as a whole can be constructed. Performing the procedure on either the multi-component system or each of the components are both feasible. For a complicated or large system, the proposed method can be performed on those elements in minimum cut sets that are constructed by fault tree analysis or Petri net model for failure [18].

Table 1
Mean value, standard deviation, and variance for different T

T (min)	μ (%)	σ (%)	σ^2 (%)
5	99.74656	0.402957	0.162374
20	99.74060	0.471612	0.222418
60	99.72771	0.652469	0.425716

6. Conclusions

An experiment of state estimation for predictive maintenance using the Kalman filter on a DC motor has been performed in this paper. The resultant prediction errors for one-step-ahead prediction are acceptable. Furthermore, the

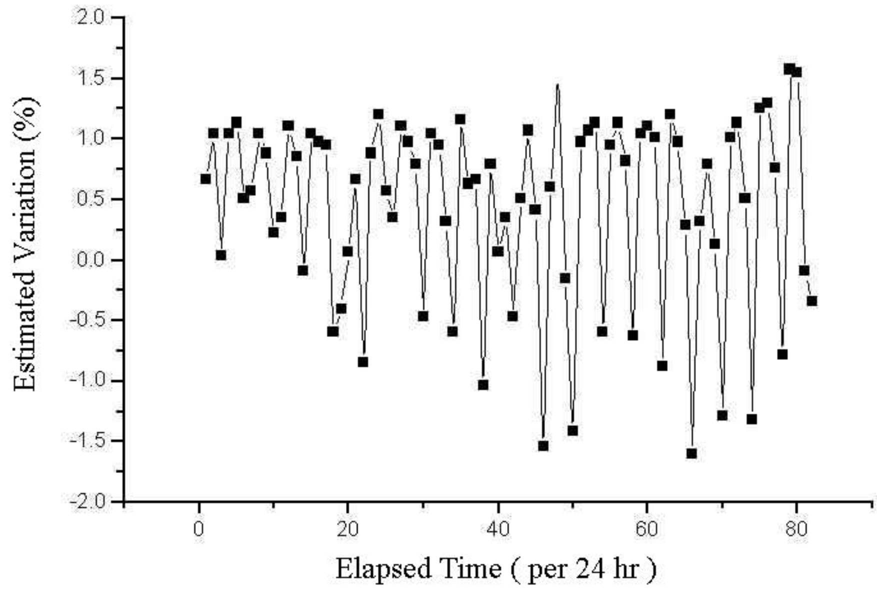


Fig. 10. Variation percentage of the estimated rotating speed.

shorter the increment time for every step in Kalman prediction uses, the higher prediction accuracy it achieves. Considerations for determining the required PM lead time and the increment time for prediction contradict to each other. How to compromise them and end up with an optimal value is important. Incorporating the proposed method with fault tree analysis or Petri net model for failure, can be performed on those elements in minimum cut sets of a complicated or large system instead of on all elements of the whole system. Failure can be prevented in time so as to promote reliability by state estimation for predictive maintenance using the Kalman filter.

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